Tutorial 8. Semi-conductors and diodes

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Zone structure of the Energy
Particle distributions with the energies
Resistance of the semi-conductors
Light Emitted Diodes (LED)

Energy of the charges in the semi-conductors

Basic distinctions between the electrical current on metals and semi-conductors are following.

- The semi-conductors do not conducts the current at a low temperature.
- The semi-conductor resistance decreases with the growth of the temperature.
- In general the resistance of the semi-conductors is higher that the conductors one.

Internal structure of the semi-conductors

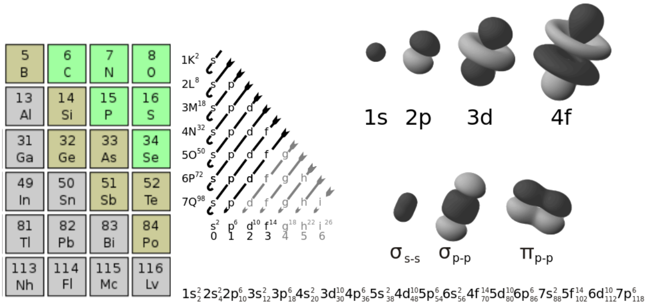
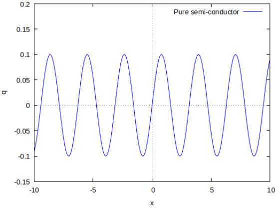


Figure: The part of the Mendeleev's periodic table and the electronic clouds.

The semi-conductors are substances which located in central columns of the Mendeleev's table.
Such substances have *S*-type and *P*-type of the electron clouds.
Physical experiments show that such structure of the electron envelope is more stable than the *S*-type one of the metal.

Zone structure



For
realizing a source of behavior for the semi-conductors let us model ones as a chain of the positive charged ions.
Then any
for electron one have a electric field with periodic stricture. The structure looks like a sequences of wholes.

Roughly speaking one can consider an periodic force acting of the charge like follows:

$$q(x) = q(x + T).$$

Where $n \in \mathbb{N}$.
A quantum mechanical point of view considers any particle as wave with an energy

$$E = h\nu,$$

here h is the Plank constant and ν is a frequency.

Differential equation for the charge onto the chain

Define a wave function of the charge by $\Psi(t, x)$.
In general positions the Schródinger equation has a form:

$$ih\partial_t \Psi = -h^2 \partial_x^2 \Psi + q(x)\Psi.$$

The de Broglie wave has frequency

$$\nu = \frac{E}{h}.$$

Therefore

$$\Psi(x, t) = e^{iEt/h}\psi(\xi), \quad x = \xi/h.$$

Such substitution gives:

$$\psi'' + (E - \tilde{q}(\xi))\psi = 0, \quad \tilde{q}(\xi) = q(\xi/h).$$

Floquet theory

Let $\phi(\xi, \xi_0, \omega)$ and $\bar{\phi}(\xi, \xi_0, \omega)$ be linear independent solutions with initial conditions:

$$\begin{pmatrix} \phi(\xi_0, \xi_0, \omega) & \bar{\phi}(\xi_0, \xi_0, \omega) \\ \phi'(\xi_0, \xi_0, \omega) & \bar{\phi}'(\xi_0, \xi_0, \omega) \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ i\omega & -i\omega \end{pmatrix}.$$

Functions ϕ and $\bar{\phi}$ define two columns of matrix $\Phi(\xi, \xi_0, \omega)$. So at $\xi + T/h$ the same solution can be written as:

$$\Phi(\xi + \tau, \xi_0, \omega) = \Phi(\xi, \xi_0, \omega)\,\hat{T}(\xi_0, \omega), \quad \tau = T/h.$$

where

$$\hat{T}(\xi_0, \omega) = \begin{pmatrix} a & \bar{b} \\ b & \bar{a} \end{pmatrix}.$$

The Wronskian is a constant and:

$$W(\Phi) = \phi\bar{\phi}' - \bar{\phi}\phi' = -2i\omega.$$

Then

$$|a|^2 - |b|^2 = 1.$$

Floquet theory

Let's consider properties of the solution

$$\psi(\xi_0, \xi_0, \omega) = 1, \quad \psi(\xi + \tau, \xi_0, \omega) = \lambda \psi(\xi, \xi_0, \omega).$$

Then

$$\psi = C\phi + D\bar{\phi}.$$

and at point $\xi_0 + \tau$ one gets:

$$C(a\phi + b\bar{\phi}) + D(\bar{b}\phi + \bar{a}\bar{\phi}) = \lambda(C\phi + D\bar{\phi}).$$

Then for coefficients of linear independent functions one gets:

$$\begin{aligned} (a - \lambda)C + \bar{b}D = 0, \\ bC + (\bar{a} - \lambda)D = 0. \end{aligned}$$

Floquet theory

Nontrivial solution exists if

$$(a-\lambda)(\bar{a}-\lambda)-|b|^2=0,$$

or

$$\lambda^2-(a+\bar{a})\lambda+|a|^2-|b|^2=0,$$

or

$$\lambda^2-2a_R\lambda+1=0,$$

here a_R is a real part of a. This equation has two solution.

$$\lambda_{12}=a_R\pm\sqrt{a_R^2-1}.$$

Floquet theory

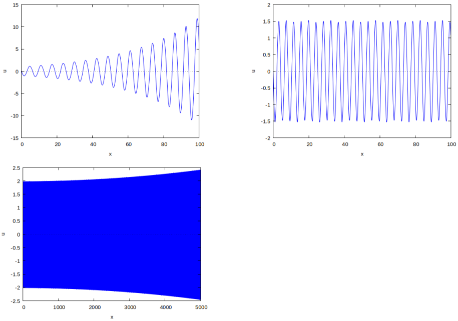


Figure: Typical growing solution as $q(x) = 0.1\sin(2x)$ and $E = 1$ and limited one as $E = 2.25$ and very slowly groving solution as $E = 4$.

Real solutions exist as $|a_R| \geq 1$.
If $a_R = \pm 1$, then $\lambda = \pm 1$ and the function ψ is periodic.
If $|a_R| > 1$, then $|\lambda| > 1$ and ψ grows as $x \to \infty$.
If $|a_R| < 1$ then $\lambda_{1,2}$ is complex.
The λ_1 and λ_2 are complex conjugated and

$$\lambda_1 \lambda_2 = 1.$$

This means $|\lambda_1| = |\lambda_2| = 1$. Hence the solution is bounded.

Zone properties of solutions of Schródinger equation

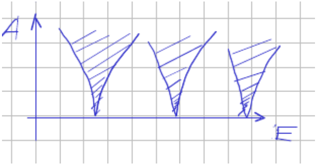


Figure: Typical values of parameters for growing solutions. For the example $q(x) = A\sin(2x)$ the tips are at the points $E = 1, 4, 9, \ldots, n^2$.

Therefore any solution $\psi(\xi + \tau)$ can be represented as $k\psi(\xi)$. Properties of the k depends of E. One can check that if

$$\sqrt{E} = \frac{2n}{T},$$

then the parameter $k > 1$. This means the solution grows exponentially. The same growth preserved for the values of E which are close to the T. Therefore one obtain a whole zone of the energy level where the solution is growth. This means the particles with such energy level accelerates go out it.

On the other hand there exist a levels of the energy where the solution have $|k| = 1$. This means the particle is stable on such energy level.

The influence of the impurity for the zone structure

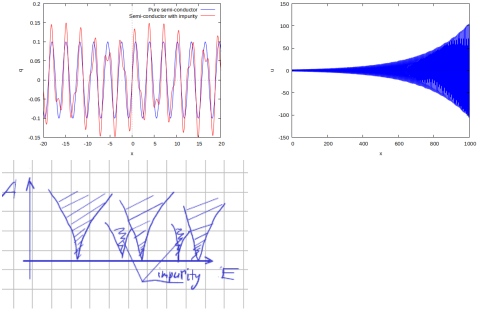


Figure: Upper figure shows a typical periodic potential with an impurity. Middle picture shows the growing solution at the impurity energy level. The lowest figure is a sketch for impurity energy level for parameters of unstable solutions.

The impurity change the periodic coefficient as the follows ($A \gg B$):

$$\psi'' + (E - A\sin(\xi) + B\sin(k\xi))\psi = 0.$$

Such changing adds stable and unstable levels of the energy.
Therefore using an impurity one can change properties of the semi-conductors.

Statistics for the electrons and wholes

Assume:

- All electrons are equivalent.
- Every level of the energy can be occupied only two electrons with opposite spins.
- Every impurity level can be occupied only one electron.

Boltzmann and Fermi-Dirac statistics

If we have n levels of the energy, then the opportunity for the distribution between the levels is $N!$.
The number of the electrons is n and these electrons era equivalent, then one must divide the numbers $N!$ on the numbers of permutations of n electrons which is $n!$.
Also one must exclude the permutations between the occupied levels, which is $(N-n)!$.
This leads to the number of permutations for n electrons onto N free energy levels:

$$W = \frac{N!}{n!(N-n)!}.$$

The most probable condition defines the equation

$$\frac{dW}{dn} = 0$$

for given numbers N and n.

Moivre-Stirling approximation

We consider a big number of the electrons, so instead of differentiating of the gamma-function:

$$n! \equiv \gamma(n+1) \equiv \int_0^\infty t^n e^{-t} dt,$$

we use the approximation of the factorial:

$$n! \sim \sqrt{2\pi n} \left(\frac{n}{e}\right)^n, \quad n \gg 1.$$

Then:

$$\log(n!) \sim n \log(n) - n + \frac{1}{2} \log(n) + \frac{1}{2} \log(2\pi).$$

$$\frac{d}{dn} \log(n!) \sim \log(n), \quad n \gg 1.$$

Formula for the derivative

$$W \sim \frac{\sqrt{2\pi N}\left(\frac{N}{e}\right)^N}{\sqrt{2\pi n}\left(\frac{n}{e}\right)^n \sqrt{2\pi(N-n)}\left(\frac{N-n}{e}\right)^{N-n}} =$$

$$\frac{1}{\sqrt{2\pi n}\left(\frac{n}{N}\right)^n} \frac{1}{\sqrt{\left(1-\frac{n}{N}\right)}\left(1-\frac{n}{N}\right)^{N-n}}.$$

Also we define the value which is called as *entropy*:

$$S = -\log(W) \sim n\log(n) + \frac{1}{2}\log(n) - n\log(N).$$

The connection between entropy and temperature:

$$\frac{dS}{dn} \sim \frac{E}{kT},$$

then

$$\log(n) - \log(N) \sim -\frac{E}{kT}, \quad \frac{n}{N} \sim e^{-\frac{E}{kT}},$$

here k is the Boltzmann constant and T is a temperature.

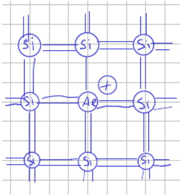
Dependence of the resistance for semi-conductors

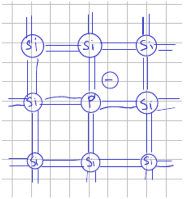
The theoretical dependency of the conductivity for given j-th level looks like:

$$\sigma = A_j e^{-\frac{E_j}{kT}},$$

here T is a temperature, k is the Boltzmann constant. E is an energy on the j-th level and A_j is some coefficient which is defined on material of the semi-conductor.

Impurity resistance





The other part of the resistance depends on the impurity. There are two type of the impurity.

- N-type impurity (donor).
  A typical example is Si with P
- P-type impurity (acceptor).
  A typical example is a Si with Al.

Two pieces of the p-type and n-type connected semi-conductor give a diode.

Physical nature of the light-emitting diode (LED)

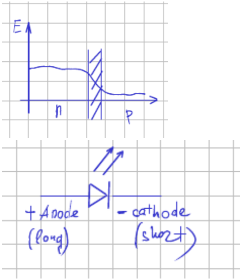


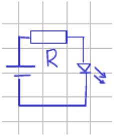
Figure: The level for energy for the charge carrier is different for the n- and p-type of semiconductors and the image of the LED of the sketches for circuits.

When the current flow thought
the diode then the electrons and holes
and as result there are emit an energy.
To be visible that frequency
of the emitted quantum should
be from interval of the visible light.
Often the flight emitted
the LED is transform due to capture
and emission by luminescent to
be appropriated for the human eyes.

LED, voltage and color

Color	Wavelength	Voltage drop	Semi-conductor meterial
RED	610-760	1.63-2.03	Al Ga As Ga As P
ORANGE	590-610	2.03-2.10	Ga As P Al Ga, In P
YELLOW	570-590	2.03-2.18	Ga As P Al Ga, In Ga P
GREEN	500-570	2.03-2.10	Al Ga, In P Ga P Al Ga P
BLUE	450-500	2.48-3.7	Zn Se In Ga, N Syntetic sapphire
PURPLE	400-450	2.76-4.0	In Ga N
WHITE	Broad spectrum	2.8-4.2	Blue/UV diode with yellow phosphor

LED circuit



To light the circuit should be provided sufficient current, but high value of current can damage the LED. Therefore
the voltage should be controlled.
The voltage drop across the LED is approximately constant and small increase of the voltage make great change of the current and light.
The value of the voltage can be controlled by serial resistance in the circuit.
The resistance one should calculate using the Ohm law:

$$R = \frac{V - V_{LED}}{I}.$$

Typical values of the V_{LED} and I can be find in the previous table.

LED Efficiency

Type	Power(W)	Lumens	Efficiency(Lm/W)	Av. Life
Incandescent	75	1200	16	1,000
CFL	27	1700	63	10,000
LED	17	1400	82	25,000

LEDs pro and contra

- Higher efficiency with respect to CFL.
- Small size.
- LEDs are well working with often on-off cycles.
- Long time and shock resistance.
- Voltage sensitivity.
- Color differ from the specter of sunlight and incandescent light.
- Single LED does not provided a spherical distribution of the light.