Tutorial 1. Electrical resistivity.

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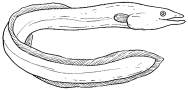
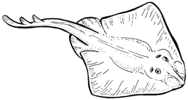
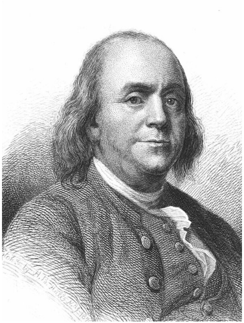
Electricity. Fishes and static electricity

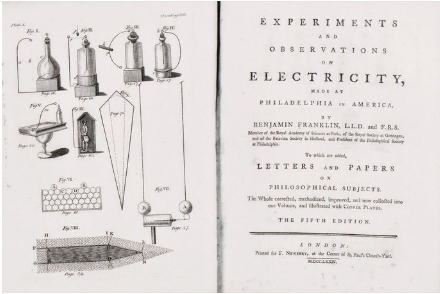




Ancient Egyptians knew about several kinds of electric fishes. These fishes have a special electric organ which can absorb and discharge up to 860V and current about 1 A. These values correspond to power similar to 1 horse force.
The static electricity was also well known since ancient times. There was a different type of electricity on the amber and wool. The static electricity is absorbed by surface of bodies and an electrical charge reaches kilovolts. So the charge caused by static electricity can destroy electronic components.

Atmospheric electricity





EXPERIMENTS
AND
OBSERVATIONS
ON
ELECTRICITY,
MADE AT
PHILADELPHIA IN AMERICA,
BY
BENJAMIN FRANKLIN, L.L.D. and F.R.S.
Member of the Royal Academy of Sciences at Paris, of the Royal Society at Gottingen, and of the Batavian Society in Holland, and Prefident of the Philofophical Society at Philadelphia.

To which are added,
LETTERS AND PAPERS
ON
PHILOSOPHICAL SUBJECTS.

The Whole corrected, methodized, improved, and now collected into one Volume, and illuftrated with COPPER PLATES.

THE FIFTH EDITION.

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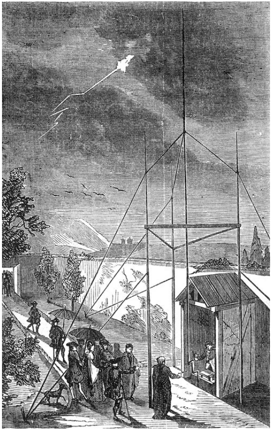
M.DCC.LXXIV.

One of the first discoverers of atmospheric electricity was Benjamin Franklin (1750) who proposed a conductor for atmospheric electricity by a kite during a thunderstorm.

Atmospheric electricity



At May 10, 1752 Thomas-François Dalibard made such an experiment with the kite.

Atmospheric electricity

Рис.: Richmann and Lomonosov

At the same time Georg Richmann together with Mikhail Lomonosov (1752-53) had experimented with Richmann's original device for measuring the atmospheric electricity.
Georg Richmann was killed by a lightning while he was experimenting with the atmospheric electricity at July 26, 1753.

Leiden jar, Galvani and Volta

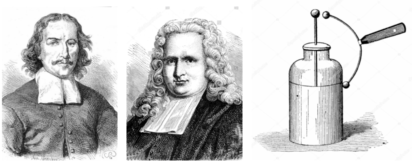


Рис.: Von Guericke and Musschenbroek

Otto von Guericke (1678) and later A.Volta (1775) used triboelectric effect to obtain a source of electricity.
There was a new gadget called to catch the electricity which was constructed by Musschenbroek (1746). It was called a leyden jar.
The leyden jar looks like a glass pot. This equipment contains two metallic joints. One joint connected with internal content of the pot and another one is a metal surface of the pot. This stuff is a image of contemporary capacitor.

Electric circuit 1746

Abbot Nollet used leyden jar for public experiments. One of famous experiment looks like circus with hand by hand people. When they touched pins of the jar they were shocking by electricity and ridiculous twitched.

Volta-Galvani element

Рис.: Galvani and Volta

At 1800 year A. Volta published his construction of the Galvani-sell which contains two metallic cylinders located into a liquid. This equipment give possibility to generate an electric current.

Galvani-Volta element

ИЗВЕСТІЕ

о

ГАЛЬВАНИ - ВОЛЬТОВСКИХЪ ОПЫТАХЪ,

которые производилЪ

ПрофессорЪ физики Василий ПетровЪ,

посредствомЪ огромной наипаче баттереи, состоявшей *иногда изЪ* 4200 медныхЪ и цинковыхЪ кружковЪ, и находящейся при Санкт – Петербургской Медико - Хирургической Академіи.

Later in 1803 V.V. Petrov construct the Galvani-Volta accumulator which was able to produce a difference of electric potentials about 1500 V. Such large difference of the electric potential allowed to study numerous electrical phenomena.

Cathode rays

In 1859 Julius Plucker studied a conductivity and observed phosphorescent light around cathode. This light was called cathode rays. It was clear that the cathode rays is a cause of electric current into the tube. In 1874 William Crookes made experiments with vacuum cathode tube. He established that the cathode rays can rotate a small paddle wheel. In 1890 experiments of Arthur Schuster allowed to establish a ratio of mass and charge of the cathode rays. The cathode rays were passed though an electric field of between two charged plates. The formula for electrical force and mechanical acceleration looks like:

$$ma = KqE, \quad \frac{q}{m} = \frac{a}{KE}.$$

Here m is mass of particle, q is the charge, a is acceleration of the particle, E is attitude of electric field and $K = 1/(4\pi\epsilon)$, where ϵ is a permittivity (диэлектрическая проницаемость среды).

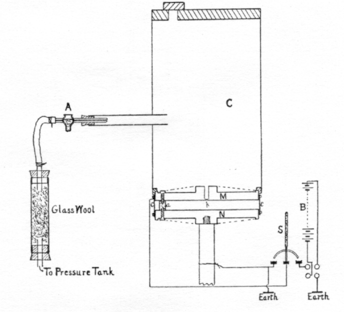
Is electron a particle?

The formula for the ratio still can be treated as a formula for a field or for some motion discontinuous substance. Later J.J. Thomson experimented with different materials and established the properties of the cathode rays do not depend on the material of the cathode and these experiments were more similar on the experiments with particles instead of an experiments with the rays.
Later he and his colleague experimented with radiation and established equivalence between particles of nuclear fusion and the cathode rays.

Is electron a particle?

However there were some reason to doubt. For example the cathode rays overcame thin aluminum foil (1892). That contradict with a concept of hard substance.
In spite on the doubts J.J. Thomson published his results and claimed that the cathode rays contained charged particles which is contained in atom. Thus the experiments with electricity shown that the atom consists from smaller particles.

Mass and charge of the electron



Finally the mass and charge of the electron were established by R.Millikan in 1909-1911 years. He organized experiments with oil charged drops.

Millikan's experiment

A speed of falling drop due to gravitation one can obtain and calculate. As result one can find attitude of mass for the drop. Behavior of the drop in electrical field depended on the charge and the experiments indicated discrete properties of the charge.
The discrete nature of the charge allowed to obtain the elementary charge. In the experiments the charged drops captured by the electric field and did not fall. In this case to get the elementary charge one can use the formula mentioned before:

$$mg = KEq, \quad q = \frac{mg}{KE},$$

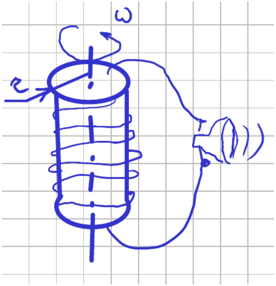
where g is gravitational acceleration. Millikan claimed that $q = 1.59 \times 10^{-19}$C. This value differs with calculated now on %0.62.

L.I. Mandelstamm and N.D. Papaleksi experiment


To define the source of electricity physicists made a lot of experiments.

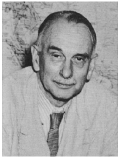
- **The first question was have the carriers of the electricity a mass?**
- The answer was done by experiment by L.I. Mandelstamm and N.D. Papaleksi in 1916. They had made an experiment with a rotating bobbin.

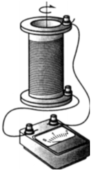
L.I. Mandelstamm and N.D. Papaleksi experiment



Let us consider an electronic circuit which contains a bobbin and register of the electricity current. When the bobbin is in a rest or rotates with a stable velocity then the current is not registered in such circuit.
If one stops the rotation of the bobbin then the short electric impulse can be detected. It means that the carriers of the current carry out the motion after the stop of the rotation. Therefore that carriers have a mass.

Tolmen and Steward's experiment





At 1919 R.Tolmen and D.Steward make the same experiment as a Mandelstamm and Papaleksi, but record the value of the electric charge which was generated in this bobbin.
Using a equivalency between the mechanical force which appears when the rotation was stopped and an electricity force which appears due to the difference of the charged they obtained:

$$qE = ma, \quad E = \frac{m}{q}a = \frac{m}{q}r\dot{\omega}.$$

Here q is a charge, E is electric field, m is mass of charged particle and $a = r\dot{\omega}$ is linear acceleration on the radius r and angle acceleration $\dot{\omega}$.

Tolmen and Steward's experiment

The electric field actions on all length of the bobbin and therefore in the bobbin produces an difference of the electric potentials (voltage) like

$$\mathcal{E} = \int_0^l E dl = \frac{m}{q} r\dot{\omega} l.$$

Such voltage inducted the current like

$$I = \frac{\mathcal{E}}{R} = \frac{m}{q}\frac{rl\dot{\omega}}{R},$$

where R is an electric resistance. Full charge which cross due to break of the bobbin rotation is equal to

$$Q = \int_0^t I dt = \int_0^t \frac{m}{q}\frac{rl\dot{\omega}}{R} dt = \frac{m}{q}\frac{rl\omega}{R}.$$

The attitudes Q, R, l, ω are known experimental data, therefore:

$$\frac{m}{q} = \frac{rl\omega}{QR}$$

The fraction (q/m) equals to the same value for the electron.

A speed of the carriers in the conductors

The electric current propagates with the same speed as the speed of the electric field. But the carriers of the charge propagate slower. There are two different mathematical models for the motion of the charge in the conductor.
The first one is based on the classical approach to the electricity and to the electrons as a classical charged particles.
Another one is based on the quantum mechanics.

Resistors

Рис.: Ohm and Ampere

George Ohm (1825) claimed that the current diminished on the electrical circuit and this property depends on the diameter of the section of the conductor and a length of it.
The electric resistance is defined by the formula

$$R = \frac{U}{I}.$$

Now the unit of the electric resistance of the conductor is called ohm in honor of G.Ohm. This unit is denoted by Ω.
The resistors is one of the most often appeared detail of the electronic circuits.

Designation and properties resistors

The most important properties for the resistors are

- Allowed interval of resistance with respect to given value. The error is normalized from manufacturer typically from 20% to 0.001%.
- Allowed values for the working temperature. Typically the diapason of the working temperatures is from about $80^{\circ}C$
- Allowed values for electric power. Typically the working power normalized by manufacturer. Typical values lie from 0.01W to 500W

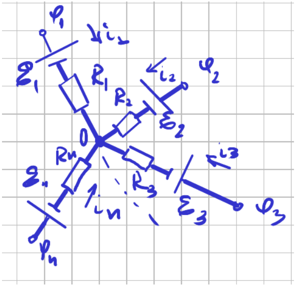
Materials for resistors

The material, which is used for given resistor, depends on the working performance. The typical materials for the resistors are

- Carbon and carbon mixture. The best for large values of the power.
- Metals and metallic oxides. The best stability with respect to temperature.
- Metallic wire. The best material for large electric current.

The resistors have different technology for production.

Kirchhoff's rules



- The sum of all currents for every given node is equal to zero.

$$\sum_k i_k = 0$$

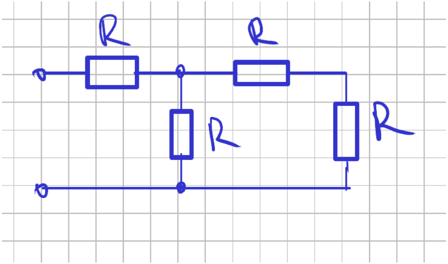
- The sum of the voltage for all circuit is equal to zero.

$$R_k i_k + \mathcal{E}_k + \phi_k = \phi_0,$$

then

$$\sum_k (R_k i_k + \mathcal{E}_k + (\phi_k - \phi_0)) = 0.$$

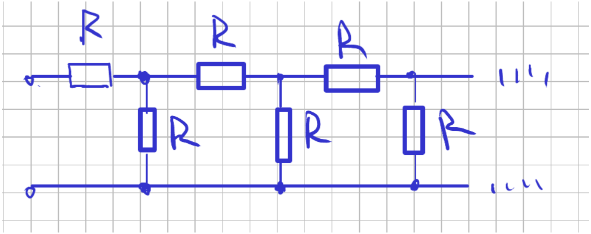
Example 1



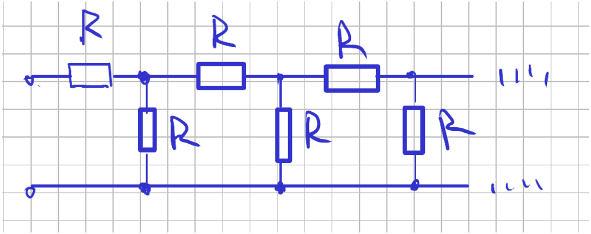
$$\frac{1}{r} = \frac{1}{R} + \frac{1}{2R} = \frac{3}{2R}, \qquad r = \frac{2}{3}R,$$

$$R = R + r = R + \frac{2}{3}R = \frac{5}{3}R.$$

Example 2

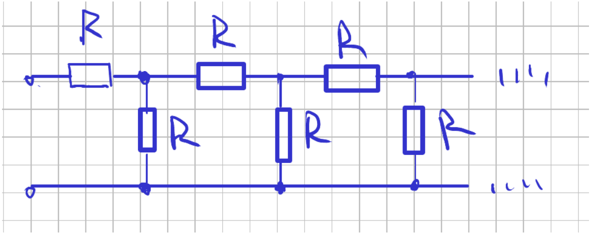


Example 2



$$\frac{1}{\rho} = \frac{1}{R} + \frac{1}{r} = \frac{r+R}{Rr}, \quad \rho = \frac{rR}{R+r},$$

Example 2



$$\frac{1}{\rho} = \frac{1}{R} + \frac{1}{r} = \frac{r+R}{Rr}, \quad \rho = \frac{rR}{R+r},$$

$$r = R + \rho, \quad r = R + \frac{rR}{R+r},$$

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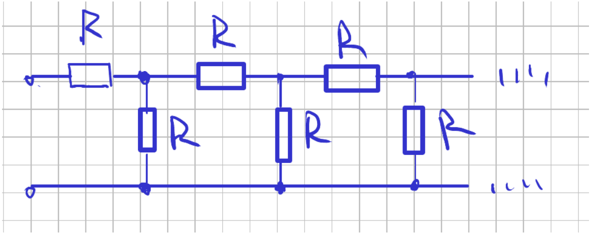
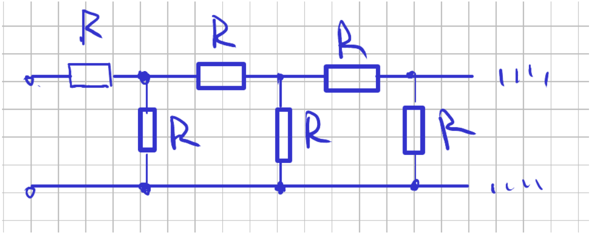


$$\frac{1}{\rho} = \frac{1}{R} + \frac{1}{r} = \frac{r+R}{Rr}, \quad \rho = \frac{rR}{R+r},$$

$$r = R + \rho, \quad r = R + \frac{rR}{R+r},$$

$$r(R+r) = R(R+r) + rR, \quad r^2 + rR = R^2 + 2Rr,$$

Example 2



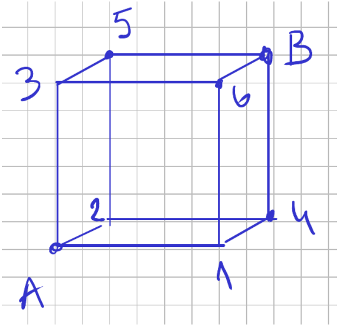
$$\frac{1}{\rho} = \frac{1}{R} + \frac{1}{r} = \frac{r+R}{Rr}, \quad \rho = \frac{rR}{R+r},$$

$$r = R + \rho, \quad r = R + \frac{rR}{R+r},$$

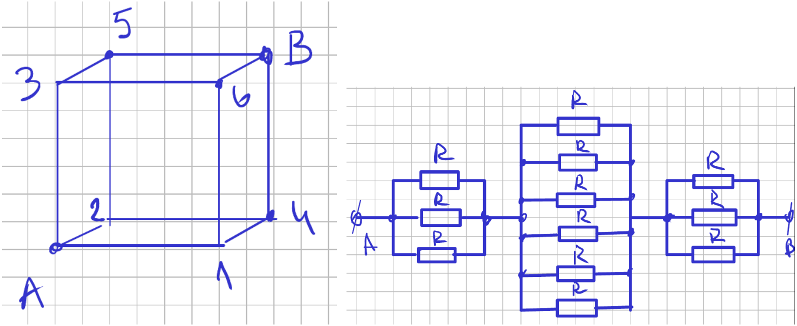
$$r(R+r) = R(R+r) + rR, \quad r^2 + rR = R^2 + 2Rr,$$

$$r^2 - Rr - R^2 = 0, \quad r = \frac{1+\sqrt{5}}{2} R.$$

example 3

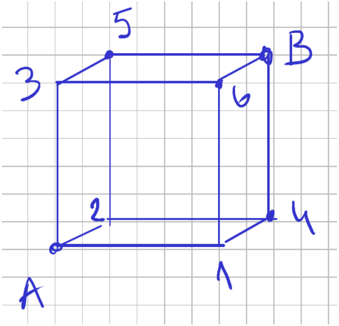


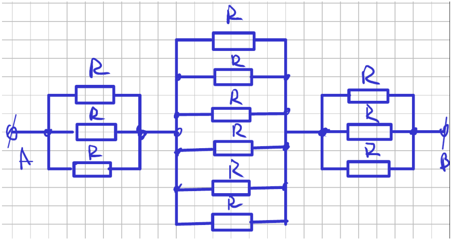
example 3



$$U_1 = U_2 = U_3, \quad U_4 = U_5 = U_6.$$

example 3

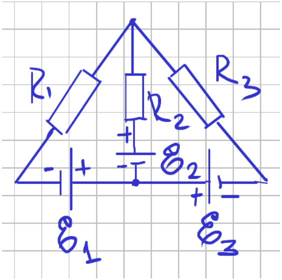




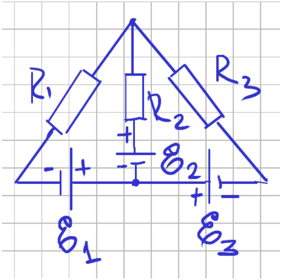
$$U_1 = U_2 = U_3, \quad U_4 = U_5 = U_6.$$

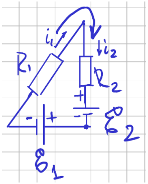
$$R_{AB} = \frac{R}{3} + \frac{R}{6} + \frac{R}{3} = \frac{5}{6}R.$$

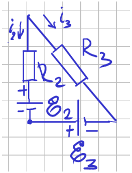
Example 4

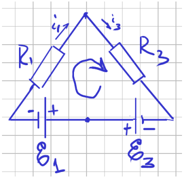


Example 4

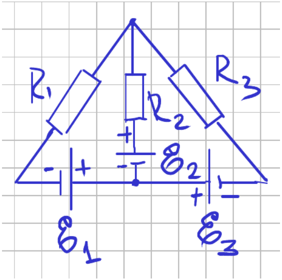


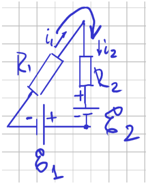


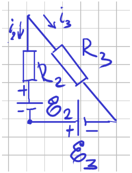


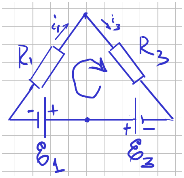


Example 4









- $R_1 i_1 + R_2 i_2 - \mathcal{E}_1 - \mathcal{E}_2 = 0$
- $-R_2 i_2 + R_3 i_3 + \mathcal{E}_2 + \mathcal{E}_3 = 0$
- $R_1 i_1 + R_3 i_3 - \mathcal{E}_1 + \mathcal{E}_3 = 0$
- $i_1 - i_2 - i_3 = 0.$

$$\begin{pmatrix} R_1 & R_2 & 0 \\ 0 & -R_2 & R_3 \\ R_1 & 0 & R_3 \end{pmatrix} \sim \begin{pmatrix} \mathcal{E}_1 & \mathcal{E}_2 & 0 \\ 0 & -\mathcal{E}_2 & -\mathcal{E}_3 \\ \mathcal{E}_1 & 0 & -\mathcal{E}_3 \end{pmatrix}$$

$$\begin{pmatrix} R_1 & R_2 & 0 \\ 0 & -R_2 & R_3 \\ R_1 & 0 & R_3 \end{pmatrix} \sim \begin{pmatrix} \mathcal{E}_1 & \mathcal{E}_2 & 0 \\ 0 & -\mathcal{E}_2 & -\mathcal{E}_3 \\ \mathcal{E}_1 & 0 & -\mathcal{E}_3 \end{pmatrix}$$

$$\begin{pmatrix} R_1 & R_2 & 0 \\ 0 & -R_2 & R_3 \\ 0 & -R_2 & R_3 \end{pmatrix} \sim \begin{pmatrix} \mathcal{E}_1 & \mathcal{E}_2 & 0 \\ 0 & -\mathcal{E}_2 & -\mathcal{E}_3 \\ 0 & -\mathcal{E}_2 & -\mathcal{E}_3 \end{pmatrix}$$

Second and third rows are linear dependent.
The linear independent system is follows:

$$\begin{aligned} R_1 i_1 + R_2 i_2 - \mathcal{E}_1 - \mathcal{E}_2 &= 0, \\ -R_2 i_2 + R_3 i_3 + \mathcal{E}_2 + \mathcal{E}_3 &= 0, \\ i_1 - i_2 - i_3 &= 0. \end{aligned}$$

Summary

- Classical experiments. Electron, mass and charge.
- Electrons as carriers of the charge in metals.
- Resistors. Resistivity.
- Ohm's law and Kirchhoff's rules for electric circuits.